Felix Hausdorff – Paul Mongré

In a review of Hausdorff's principal work, "Grundzüge der Mengenlehre" (1914), published in 1921 by the American mathematician Henry Blumberg, one reads: *It would be difficult to name a volume in any field of mathematics, even in the unclouded domain of number theory, that surpasses the Grundzüge in clearness and precision.*[1]
This statement might be compared with another from a letter written by Paul Lauterbach, writer and translator, to the musician and Nietzsche scholar, Heinrich Köselitz (Pseudonym: Peter Gast). There we read the following in reference to Hausdorff: *A Dionysian mathematician! That sounds incredible; but let him send something to you and we will wager that there is something about him to be experienced.*[2]
The expression „Dionysian“ refers to Dionysius, the Greek god of wine, fertility, but also of ecstasy and the intoxicating, irrational, ecstatic elements necessary for experiencing the world or the creative process. An individual who writes books of mathematics of such unsurpassed clarity and precision, on the one hand, and is considered to be Dionysian, on the other, surely must lead a remarkable double

[1] [Bl 1921], p. 116.
[2] Paul Lauterbach: Letter to Heinrich Köselitz from 30 December 1893. Goethe- und Schillerarchiv Weimar, 102/417.

existence – and just such a man was Hausdorff. As Felix Hausdorff he was an important mathematician whose work has remained relevant and influential up to the present day; as Paul Mongré he was a man of letters, a philosopher, and a critical essayist, a figure whom the journalist Paul Fechter recalled in 1948 in his autobiography, "Menschen und Zeiten", as „one of the most remarkable individuals to appear in the first decades of the twentieth century" and who „has wrongfully been forgotten by the younger generation." [3] Naturally, in this double life there were many visible and invisible threads that became intertangled and which must be retraced if one is to understand properly the man and his work. Felix Hausdorff was born in Breslau on November 8, 1868. His father, a Jewish businessman named Louis Hausdorff (1843–1896), moved in the fall of 1870 with his young family to Leipzig, where he managed various companies including linen and cotton shops. He was an educated man who already at age 13 had obtained the Morenu-Title.[4] There are several papers penned by him, among them a longer paper on the Aramaic translation of the Bible from the perspective of the Talmudic Law which appeared in the „Monatsschrift für Geschichte und Wissenschaft des Judenthums". For many years Louis Hausdorff was

[3] [Fe 1948], p. 156.

[4] Morenu being Hebrew for „our teacher"; this title was conferred on those who qualified to teach as rabbis.

involved with the „Deutsch-Israelitischen Gemeindebund“.[5] He was even brought in as a member of the executive committee of the Gemeindebund because its presiding officer thought it would be desirable *to have the decidedly conservative position, for which Mr. Louis Hausdorff was known, to be represented on the committee.*[6] In an 1896 obituary of the Gemeindebund the following was written about Louis Hausdorff: *His great and noble heart beat warmly for the affairs of his fellow believers. At the same time, he was a devoted, self-sacrificing father in the true Jewish sense; in the same manner, his bountiful acts of charity corresponded to the most beautiful tradition of our people.*[7]
Hausdorff's mother Hedwig (1848–1902) (called Johanna is various documents) was a member of the widely dispersed Jewish family Tietz. From one branch of this family came Hermann Tietz, the founder of the first department store and later the princi-

[5]This organization was founded after the creation of the German Reich to represent the interests of German Jews within the new state.

[6]Mittheilungen des Deutsch-Israelitischen Gemeindebundes, Nr. 5 (1878). The conservatives maintained a strict line with respect to conventional religious practices. They fought, for example, to have Jewish pupils freed from attending the Gymansium on the Sabbath so that they could attend the Synagogue, or at a minimum that they be freed from writing tests on these days.

[7]Ibid., Nr. 44 (1896).

pal owner of a chain of department stores „Hermann Tietz“ . During the period of the National-Socialist dictatorship the firm was „aryanized “ under the name HERTIE.

We do not know how Felix Hausdorff was reared as a child. We can only guess that he had a strict religious upbringing. In a report to the executive committee of the „Deutsch-Israelitischen Gemeindebund“ his father said the following: *The center of Judaism is not found in the sermon, nor in the religious services. Its true focus is much more to be found in the religious life of the family.*

We can only draw indirect conclusions about how Felix Hausdorff reacted to his upbringing. In one of his aphorisms, he later wrote: *Whoever invented the fable of the happiness of childhood forgot three things: religion, upbringing, and the early phases of sexuality.*[8]

In another one of his aphorisms he writes the following in regard to the rearing of children in his day: *But the method is still the same today: exterminate, hinder, cut off, deny, restrict, prohibit – it was a fundamentally negative, privatistic, prohibitive method or rearing, improving, punishing – eradicating instead of creating, amputating instead of healing.*[9]

In regard to Felix Hausdorff's religious training, the results were the opposite of that which his father

[8] [H 1897a], p. 254.

[9] [H 1897a], p. 62.

wanted to achieve: Hausdorff gave up practicing the Jewish faith. He was an agnostic who critically disputed the tenets of Jewish religion just as he did with the Christian. Still, he was never baptized, a religious rite that would have offered him considerable advantages.

Let us now turn to Hausdorff's educational background. For three years he attended the former second Bürgerschule in Leipzig; afterward, beginning in 1878, he went to the Nicolai Gymnasium in Leipzig. This school had an excellent reputation as a humanistic educational institution. Hausdorff was an outstanding pupil, the best in his class over many years, and he often was given the honor of reading the poems he had composed in Latin or German during school vacations. In his graduating class of 1887 he was the only pupil to receive the cummulative grade of „I" . The focus of the gymnasium education was on classical languages, which comprised approximately 45 per cent of the obligatory curriculum. Hausdorff was required, for example, in the final examination for graduation to write a Latin essay on the theme: „Cupidius quam verius Cicero dicit res urbanas bellicis rebus anteponendas esse" (freely translated: „it corresponds more to Cicero's interests than the truth when he states that matters of public welfare have priority over those of warfare").[10] The

[10] [JN 1887], pp. X–XI.

choice of field for his university studies may well have been a difficult one for the multi-talented Felix Hausdorff. Magda Dierkesmann, a student in Bonn from 1926–1932 who was often a guest in Hausdorff's home, reported many years later: *His versatile musical talent was so great that it was only due to the urging of his father that he gave up his plans to study music and become a composer.*[11]

By the time he graduated the decision had been reached (though we do not know what prompted it): in the annual report of the Nicolai Gymnasium for 1887 next to the list of graduates one finds a column giving the „future field of study," which for Felix Hausdorff was „natural sciences."[12]

From the summer semester of 1887 to the summer semester of 1891 Hausdorff studied mathematics and astronomy, mainly in Leipzig, but with interruptions of one semester each to study in Freiburg (SS 1888) und in Berlin (WS 1888/1889). His extant academic certificates[13] (which have survived except for the semester he spent in Freiburg) show that the student Felix Hausdorff was a young man with ex-

[11][D 1967], pp. 51–52. In a conversation with Egbert Brieskorn Frau Dierkesmann assured him that she was told this directly by Hausdorff.

[12][JN 1887], p. XVI.

[13]UA Leipzig, Film Nr. 60 und Nr. 67; Archiv der Humboldt-Universität Berlin, Univ.-Registratur, Littr. A, N. 6, Vol. 876, No. 28. For the latter information I thank Mr. Girlich (Leipzig).

ceptionally broad interests. Alongside the courses he took in mathematics, astronomy, physics, chemistry, and geography, he also attended lecture courses in philosophy and history of philosophy, languages and literatures, and on the history of socialism and the labor movement. In addition to this, he took part in a course on the scientific foundations of belief in a personal God and in another on the relationship between mental disorders and crime. In Leipzig he also attended lectures by the musicologist Paul on the history of music. His early love for music accompanied him throughout his life; numerous participants left reports of the fascinating musical evenings in his home with Hausdorff at the piano. Already as a student in Leipzig he had a special affinity for and excellent knowledge of the music of Richard Wagner.

In the final semesters of his studies Hausdorff worked closely with Heinrich Bruns (1848–1919), who was professor of astronomy and the director of the observatory at Leipzig University. Bruns, a student of Weierstrass, was above all known for his work on the three-body problem and on optics (Bruns' Eikonal). Hausdorff took his doctorate under him in 1891 with a dissertation on the refraction of light in the atmosphere.[14] This was followed by two further publications on the same subject leading up to Hausdorff's Habilitation for which he submitted a study on the

[14] [H 1891].

extinction of light in the atmosphere.[15] These early astronomical works by Hausdorff were – their excellent mathematical presentation notwithstanding – of no further consequence. Firstly, it turned out that Bruns' principal idea was unworkable (astronomical observations of refraction near the horizon were required, which, as Julius Bauschinger soon thereafter showed, were impossible to obtain with the exactitude necessary). Secondly, the new possibility for making direct measurements of atmospheric data by means of test balloons made the difficult calculations of these data that utilized refraction observations obsolete. During the period between his doctorate and his Habilitation Hausdorff completed his year of military service and he also worked for two years doing calculational work at the Leipzig observatory.

With his Habilitation Hausdorff began his career as a Privatdozent in Leipzig during which he offered a wide range of courses in various areas of mathematics. Alongside teaching and research, he also pursued his literary and philosophical interests. With his diverse interests, broad education, and cultivated sensitivities in all matters of thought, feeling, and experience, Hausdorff was drawn to a circle of noteworthy writers, artists, and publishers that included Hermann Conradi, Richard Dehmel, Otto Erich Hartleben, Gustav Kirstein, Max Klinger, Max Re-

[15][H 1895].

ger and Frank Wedekind. During the period from 1897 to 1904 – the high point of his own literary and philosophical creativity – he published eighteen of the twenty-two works that appeared under his pseudonym, including a volume of poems, a play, a book on epistemology, and a volume of aphorisms.

The book of aphorisms was the first work that Hausdorff wrote as Paul Mongré. He entitled it "Sant' Ilario. Gedanken aus der Landschaft Zarathustras".[16] Already his choice of pseudonym suggested the author's orientation: à mon gré – after my own taste. This reflected an individuality, spiritual autonomy, and a rejection of prejudices and conformity in political, social, religious, or other spheres of human affairs. The subtitle of his "Sant' Ilario", „Gedanken aus der Landschaft Zarthustras" stems from the circumstance that Hausdorff completed his book while recuperating on the Ligurian coast near Genoa, the same locale where Friedrich Nietzsche wrote the first two parts of "Also sprach Zarathustra"; the subtitle also naturally suggests the spiritual affinity to Nietzsche. In a preview of "Sant' Ilario" in the weekly magazine "Die Zukunft" Hausdorff explicitly acknowledged his debt to Nietzsche: *On this blissful coast [...] I followed the lonely paths of Zarathustra's creator – wonderful, narrow paths along banks and cliffs that have no room for moving an army. If one should*

[16] [H 1897a].

thus wish to count me among Nietzsche's followers, then let this serve as my own confession.[17] Hausdorff did not attempt to copy Nietzsche let alone surpass him; as one reviewer put it, there is „not a trace of mimicking Nietzsche". He positions himself, so to speak, next to him in an effort to release his individual thoughts and to gain the freedom needed to question conventional norms. Hausdorff maintained a critical distance to Nietzsche's later works. In his essay on Nietzsche's "Der Wille zur Macht", a book compiled from various fragments in the Nietzsche-Archiv, he wrote: *Nietzsche glows like a fanatic. If his moral order based on breeding were to be established drawing on our modern knowledge of biology and physiology, that could lead to a world historical scandal compared to which the Inquisition and witch trials would appear like merely harmless confusions.*[18] Yet Hausdorff took his critical standard from the younger Nietzsche, *from the gracious, moderate, understanding free spirit Nietzsche and from the cool, dogma-free, systemless skeptic Nietzsche [. . .]*[19]

Any attempt to describe the contents of a volume of aphorisms clearly makes no sense, but in order to say at least something about it, one can point

[17] [H 1897b], p. 361. For details on Hausdorff's relationship to Nietzsche, see [St 2002] as well as the historical introduction to [H 2004].

[18] [H 1902], p. 1336.

[19] Ibid., p. 1338.

to two ideas which he thematizes over and again: first, he expresses a deep skepticism with regard to all forms of teleology and, even more, ideologies or theories for improving the world that claim to know the true meaning and purpose of humanity. As exemplars of this, consider these two excerpts from the first and third Aphorisms: *The world is so full of outrageous nonsense, cracks, fragmentation, chaos, „free will"; I envy those whose good, synthesizing eyes are able to see the world as the unfolding of an „idea", a single idea.*[20] *If not truth itself, then surely the belief in holding truth is to a dangerous degree antagonistic to life and murderous for the future. Not one of those who deluded themselves that they were blessed with the truth hesitated for a moment to pronouce the grand finale, or the great day, or some other end point, turning point, or climax for humanity, and every time this meant that all future humanity was to be molded by their image, their stamp, and their narrowness.*[21] This raises the question of the relationship between the individual and society. For Hausdorff, as for Nietzsche, the individual is no mere figure within an historical process which subordinates his individuality to a higher order. On the contrary, individuals, especially those who are creative, should be placed in the center and their rights should be defended. Here, to this

[20] [H 1897a], p. 4.
[21] [H 1897a], p. 6.

point, is an excerpt from Aphorism 35: *Fruitful is anyone who calls something his own, whether making or enjoying, in speech or gesture, in longing or posessing, in science or culture; fruitful is everything that occurs less than twice, every tree growing in its soil and reaching up to its sky, every smile that belongs to only one face, every thought that is only once right, every experience that breathes forth the heart-strengthening smell of the individual!*[22]

The year 1898 saw the appearance of Hausdorff's critical epistemological study – again under the pseudonym Paul Mongré – "Das Chaos in kosmischer Auslese (Chaos in cosmic selection)". Its critique of metaphysics resulted from Hausdorff's effort to come to terms with Nietzsche's idea of eternal recurrence. His aim is nothing less than to destroy permanently *every* type of metaphysics. Regarding the world in itself, a *transcendental world core* as Hausdorff calls it, we know nothing and can know nothing. We must take „the world in itself" to be undetermined and indeterminant, a mere chaos. Our world of experience, our cosmos, is a result of selection, which we have always involuntarily undertaken and continue to undertake according to our possibilities of knowledge. Starting from that chaos there are any number of other orders, other cosmoi, that could be conceived, but from the world of our cosmos

[22][H 1897a], p. 37.

there is no possibility of drawing conclusions regarding a transcendental world. Hausdorff formulated his program in the following way: *We will have to show the full diversity of both worlds and the untenability of any reasoning from empirical consequences to transcendental premises [...] and to do so in a comprehensive generality that also goes beyond the result of Kant in a practical way [...]*[23] He describes the methodology he proposes to use to establish this claim as follows: *[...] we have [...] simply to determine those transcendental variations that leave a given empirical phenomenon unchanged.*[24]

In "Chaos in kosmischer Auslese" he attempted to carry out this program for the categories of time and space. To gain an impression of how Hausdorff applied this methodology to space, consider the following passage from his Leipzig inaugural lecture "Das Raumproblem". His argument here is based on the fact that by studying a map one can never determine the form of the original space without knowing the method of projection used to obtain it. From here he argues further that: *[...] our empirical space is just such a physical map, an image of the absolute space [absolute in the sense of transcendental]; but [...] we do not know the method of projection and so we cannot know the original. The two spaces are related by means of an unknown and undeter-*

[23] [H 1898], p. 4.
[24] [H 1898], p. 9.

mined correspondence, a completely arbitrary point transformation. Still, the empirical space maintains its value as a means of orientation; we are able to find our way with this map and we can communicate with those who also possess this map; the distortion never enters our consciousness because not only the objects but we ourselves and our measuring instruments are uniformly affected by this. [...]
If this viewpoint is correct, then it must be possible for the preimage to undergo an arbitrary transformation without changing the image: [...][25] The simplest such transformation would be a uniform shrinking or expanding of the transcendental space by a constant factor. Yet Hausdorff was concerned with *arbitrary* transformations, which means that the transcendental space must remain completely undetermined and indeterminant – such a space is thus a senseless concept, scientifically speaking. Hausdorff worked intensively on the space problem for many years; in the winter semester 1903/04 he offe-

[25][H 1903], p. 15. How such transformations might affect physical properties remains open here. In a posthumous (unfortunately undated) fragment „Transformationsprincip“ Hausdorff wrote about this: „That the physical content also might take part in the transformation needs to be considered more carefully. That is perhaps not so simple. Perhaps in this respect the principle is even objectionable – an idea I find attractive now that I've noticed that others (Poincaré) have taken up this principle!!! “ (NL Hausdorff: Kapsel 49: Fasz. 1079, Bl. 3.)

red a lecture course in Leipzig on “Zeit und Raum (Time and Space)”[26], in which he spoke of his passion for this problem. The fundamental concept of a topological space which he later created was conceived in order to accomodate practically every situation in which „spatiality,“ in the sense of neighborhoodness, plays a role. This concept was probably influenced by his philosophical reflections on the space problem.

It is especially striking that in “Chaos in kosmischer Auslese”, a philosophical study, Hausdorff brought in elements from the very newest mathematics, namely set theory. This surely unique, but also problematic aspect made the work’s reception more difficult.

In 1904 the periodical “Die neue Rundschau” published Hausdorff’s once-act play “Der Arzt seiner Ehre (The Surgeon of his Honour)”. This earthy satire dealt with duelling and the conventional code of honor of aristocrats and the Prussian officers’ corps. Such forms of chivalry had begun to appear more and more outmoded in bourgeois society. In a review that appeared in the “Hamburger Echo” on 15 November, 1904 one finds this summary opinion: *Mongré has the courage to show duelling in the light that it deserves. He treats it as comedy, about which one can agree over a glass of wine so long*

[26] NL Hausdorff: Kapsel 24: Fasz. 71.

as one is not chained like a vain fool to the fashion demon of „honor“. “Der Arzt seiner Ehre” was Hausdorff’s greatest literary success. Between 1904 and 1912 it was performed over 300 times on stages in Berlin, Brunswick, Bremen, Breslau, Bromberg, Budapest, Dusseldorf, Dortmund, Elberfeld, Elbing, Frankfurt, Furth, Graz, Hamburg, Hannover, Kassel, Cologne, Koenigsberg, Krefeld, Leipzig, Magdeburg, Muhlhausen, Munich, Nuremberg, Prague, Riga, Strassburg, Stuttgart, Wien, Wiesbaden and Zurich.[27] Hausdorff’s reputation as an important playwright can be judged from the banquet he attended on 18 June, 1912 at the Hotel Esplanade in Berlin held in honor of Frank Wedekind: he arrived in the company of Max Reinhardt, Felix Holländer and Arthur Kahane, the crème de la crème from the Berlin theatrical scene.[28]

We must content ourselves with these few glimpses of Hausdorff’s literary and philosophical works without touching on his poetry volume “Ekstasen” (1900) or his essays, true pearls in this literary genre.[29] Most of the essays appeared in the periodical „Neue Deutsche Rundschau (Freie Bühne)“ (later

[27] For the review cited above and the information about these performances I thank U. Roth, Munich.

[28] Frank Wedekind: “Gesammelte Briefe”. Hrsg. von Fritz Strich. Bd.2, München 1924, p. 269; for this reference I thank Ariane Martin, Mainz.

[29] On this, see [V 2000].

renamed „Die neue Rundschau (Freie Bühne)"), the then leading literary journal about which was said: „Remember, that your life will pass, even if you made it into the Neue Rundschau (Gedenke, Mensch, dass Du vergehst, auch wenn Du in den Neuen Rundschau stehst)".

After the Second World War Hausdorff's philosophical writings were for a long time forgotten, and the same holds true for his literary works. One might conjecture that anti-Semitism and the cultural barbarism of the Nazi distatorship contributed to this neglect. Up until then there was still a public awareness of Hausdorff as a philosopher and writer, as can be seen from this entry in the 1931 edition of the Großen Brockhaus[30]: *Hausdorff, Felix, Mathematician and Writer. He is the author of: „Grundzüge der Mengenlehre" (1914), and under the pseudonym Paul Mongré the epistemological study „Das Chaos in kosmischer Auslese" (1898), the works „Sant' Ilario. Gedanken aus der Landschaft Zarathustras" (1897) and „Ekstasen" (1900). H. has close affinities to the fundamental ideas of Nietzsche; he rejects all metaphysics and regards the world of experience as a segment drawn by consciousness out of a lawless chaos.* Hausdorff's Gesammelte Werke[31] contain all of his philosophical and literary works along

[30] Dates and localities of his life have been omitted.

[31] More about this edition can be found at the conclusion of this article.

with detailed commentary. The editors of these volumes have in many places emphasized that this part of Hausdorff's creative work been wrongly neglected. Regarding his philsophical contributions, Werner Stegmaier had this to say in the preface to volume VII: *The more I delve into Felix Hausdorff's writings, the more they command my respect: for their clarity, their honesty, their noble modesty, their intellectual independence, and above all for their astonishing present-day relevance. Perhaps now the time has come, after one hundred years, that they can lead to fruitful philosophical orientation as they so deserve.*[32]

After a few short biographical remarks we want to turn to the mathematical works. In 1899 Hausdorff married Charlotte Goldschmidt, the daughter of the Jewish physician, Siegismund Goldschmidt, from Bad Reichenhall. His stepmother, incidentally, was the famous feminist and preschool pedagogue, Henriette Goldschmidt. In 1900 the Hausdorffs' only child, their daughter Lenore (Nora), was born; she survived through the Nazi era and died at a ripe old age in 1991 in Bonn.

In December of 1901 Hausdorff was appointed as an unofficial associate professor (außerplanmäßiger Extraordinarius) at Leipzig University. In submitting the faculty's proposal for Hausdorff's appointment,

[32] [H 2004], p. VII.

which contained a very favorable assessment given by his colleagues and composed by Heinrich Bruns, the Dean added the following remark: *The faculty considers itself, however, duty bound to inform the Royal Ministry that the present proposal was not approved by all members in the meeting on the 2nd of November this year, but rather by a vote of 22 to 7. The minority who voted against Dr. Hausdorff did so because he is of the Jewish faith.*[33]This amendatory remark illuminates at a glance the open anti-Semitism that was especially on the rise across the entire German Empire after the financial crash (der Gründerkrach) that followed its founding in 1871. Leipzig was at the center of the anti-Semitic movement, in which students played a large role. This may well have been one reason why Hausdorff never felt particularly comfortable teaching there; another reason was the strong sense of hierarchy among the full professors (Ordinarien), who tended to disregard their junior colleagues. Later in Bonn Hausdorff commented retrospectively in a letter to Friedrich Engel: *In Bonn one has the feeling, even as a junior faculty member (Nicht-Ordinarius), of being formally accepted, a sense I could never bring myself to feel in Leipzig [an der Pleisse].*[34]

[33] Archiv der Universität Leipzig, PA 547. The full report is reproduced in [BP 1987], pp. 231–234.

[34] Letter from 21. February 1911. NL Engel, UB Gießen, Handschriftenabteilung.

After his Habilitation Hausdorff wrote papers on optics ([H 1896]), non-Euclidean geometry ([H 1899]), hypercomplex number systems ([H 1900b]), insurance mathematics ([H 1897c]), and probability theory ([H 1901b]). The last two works contain several noteworthy results that were not without influence. In [H 1897c] Hausdorff introduced the variance of an insurer's losses as a measure of risk. Whereas today theories of individual risk have given way to collective risk theories, nevertheless variance of loss remains a fundamental quantity for the evaluation of insurance plans with fixed coverages and premiums. In this paper Hausdorff also presented a first correct proof the the Theorem of Hattendorff. For various types of life insurance he calculated the variance of loss, results which were taken up immediately afterward in the textbook literature. In [H 1901b] Hausdorff called special attention to the concept of conditional probability, a notion of fundamental importance that had only been used implicitly up until then. He also introduced new terminology („relative probability") along with a suitable notation for it.[35] In this same paper (and independent of Thiele) he dealt with semi-invariants and gave highly simplified derivations of the Gram-Charlier series of Type A. His example of a sequence of independent, identi-

[35]Kolmogoroff later adopted this notation ($P_B(A)$) in his book "Grundbegriffe der Wahrscheinlichkeitsrechnung" (1933).

cally distributed random variables $X_1, X_2, \ldots$ with density $\varphi(x) = \frac{1}{2}\, e^{-|x|}$, for which

$$Z_n = \sum_{k=1}^{n} a_k X_k \quad \text{with} \quad a_k = \frac{1}{(k+\frac{1}{2})\pi}$$

does not converge to a normal distribution provided the motivation for Paul Lévy to formulate an interesting conjecture about the decomposition of the normal distribution into two independent components. This conjecture from the early 1930s was proved in 1936 by Harald Cramér.[36]

Hausdorff's principal field of research, however, soon became set theory, especially the theory of ordered sets. Initially it was his philosophical interests that led him to begin studying Cantor's ideas.[37] Already in the summer semester of 1901 Hausdorff offered a lecture course on set theory; this was nearly a first in Germany, only Ernst Zermelo's course in Göttingen the previous semester preceded it. (Cantor himself never offered lectures on set theory in Halle.) It was in the context of teaching this course that Hausdorff made his first discovery in set theory: the type class $T(\aleph_0)$ of all countable order types has the power $\aleph$ of the continuum. He soon found, though, that this theorem was already in Felix Bernstein's Dissertation – as Hausdorff carefully noted in the margin of

[36] For details, see [H 2005], pp. 579–583.
[37] See [H 2002], pp. 3–5.

manuscript: *Presented on 27 June 1901. Dissertation of F. Bernstein received on 29 June 1901.*[38]

Hausdorff engaged in a thorough study of ordered sets, motivated in large part by Cantor's continuum problem, which poses the question of finding the place occupied by $\aleph = 2^{\aleph_0}$ in the sequence of the $\aleph_\alpha$.[39] In a letter to Hilbert from 29 September 1904, he revealed that this problem „had plagued me almost like an obsession".[40] He thought that the theorem $\mathrm{card}(T(\aleph_0)) = \aleph$ offered a new strategy for attacking the problem. Cantor had long conjectured that $\aleph = \aleph_1$, but it had only been proved that $\aleph \geq \aleph_1$, where $\aleph_1$ represents the „number" of possible well-orderings of a countable set. It turned out that $\aleph$ is the „number" of *all* possible orderings of such a set, which naturally led to the study of orderings that were more general than well-orderings but more special than arbitrary orderings. This was precisely what Hausdorff did in his first set-theoretic publication from 1901[41] in which he studied „graded sets" (gestufte Mengen). In the meantime we know from the results of Kurt Gödel and Paul Cohen that

[38] NL Hausdorff: Kapsel 03: Fasz. 12, Bl. 37.

[39] Hausdorff's reflections on time as background for his study of order structures are taken up by Erhard Scholz in his article "Logische Ordnungen im Chaos: Hausdorffs frühe Beiträge zur Mengenlehre" (in [Br 1996], pp. 107–134).

[40] Niedersächsische Staats- und Universitätsbibliothek zu Göttingen, Handschriftenabteilung, NL Hilbert, Nr. 136.

[41] [H 1901a].

this strategy for solving the continuum problem had no more chance of attaining its goal that did Cantor's approach, which tried to generalize the Cantor-Bendixson theorem for closed sets to the case of arbitrary uncountable point sets.

In 1904 Hausdorff published the recursion formula that now carries his name: for every nonlimit ordinal μ

$$\aleph_{\mu}^{\aleph_\alpha} = \aleph_\mu \, \aleph_{\mu-1}^{\aleph_\alpha}.$$

This formula, together with the concept of cofinality that Hausdorff later introduced, served as the foundation for all further results on the exponentiation of alephs. Hausdorff's precise knowledge of the problematics of recursion formulae of this type enabled him to detect an error in Julius König's lecture presentation at the 1904 International Congress of Mathematicians held in Heidelberg. König claimed to have „proved" that the continuum cannot be well-ordered, which would have implied that its cardinality is not an aleph, a result that evoked considerable interest.[42]

During the period from 1906 to 1909 Hausdorff pu-

[42] Determining that it was Hausdorff who uncovered this error is particularly significant in view of the fact that for well over fifty years the historical literature has drawn a faulty picture of the events in Heidelberg; detailed information can be found in [H 2002], pp. 9–12 and in [Pu 2004]. Further important source material on this story can be found in [Eb 2007].

blished his fundamental works on ordered sets.[43] Only a few points concerning these studies can be touched on here. Of fundamental importance for the entire theory is Hausdorff's concepts of cofinality and coinitiality: if A is an ordered set and $M \subset A$, then A is said to be cofinal (coinitial) with M if for every $a \in A$ there exists an $m \in M$ such that $m \geq a$ ($m \leq a$). Thus, for example, (0,1) is cofinal with $\left\{\frac{m-1}{m}\right\}_{m\in\mathbb{N}}$ and coinitial with $\left\{\frac{1}{n}\right\}_{n\in\mathbb{N}}$. This concept carries over to order types: for example, the type λ associated with the set of real numbers under its natural ordering is cofinal with the type ω of the natural numbers.

An ordinal number is called regular if it is not cofinal with a smaller ordinal number, otherwise it is called singular. Hausdorff named the smallest number in each of Cantor's number classes an initial number (Anfangszahl)[44]: $\omega_0, \omega_1, \omega_2, \ldots, \omega_\omega, \omega_{\omega+1}, \ldots$ All $\omega_{\alpha+1}$ are regular, but $\omega_\omega = \lim_n \omega_n$ is cofinal with ω and thus an example of a singular initial number. Hausdorff asked whether there exist regular initial numbers with a limit number as index, a query that served as the point of departure for the theory of inaccessible cardinal numbers. Hausdorff indeed realized that, were such numbers to exist,

[43] [H 1906, 1907a, 1907b, 1908, 1909].

[44] Today these are identified with the cardinal numbers: $\aleph_0, \aleph_1, \aleph_2, \ldots, \aleph_\omega, \aleph_{\omega+1}, \ldots$.

they would have to be of „exorbitant size“.[45]
The following theorem of Hausdorff's is of fundamental importance: for every dense ordered set A without boundary there exist two uniquely determined regular initial numbers ω_ξ, ω_η such that A is cofinal with ω_ξ and coinitial with ω_η^* (where * signifies the inverse ordering). This theorem offers a sensitive instrument for characterizing gaps and elements in ordered sets, as Hausdorff showed. Following his technique, if for example the ordered decomposition $A = P + Q$ represents a gap, that is P has no greatest and Q no smallest element, then according to the above theorem there exist two uniquely determined regular initial numbers ω_ξ, ω_η such that P is cofinal with ω_ξ and Q is coinitial with ω_η^*. Hausdorff calls the pair $(\omega_\xi, \omega_\eta^*) =: c_{\xi\eta}$ the character of the gap. By this means one obtains from the decomposition $A = P + \{a\} + Q$ a uniquely determined character for the element a, though here one must allow for characters of the type $(1, \omega_\eta^*)$, $(\omega_\xi, 1)$ or $(1, 1)$. Thus, in the set of rational numbers (with the natural ordering) all gaps and elements have the character c_{00}.
If W is a given set of characters (for elements and gaps), for example $W = \{c_{00}, c_{01}, c_{10}, c_{22}\}$, the question arises whether there exists an ordered set having precisely W as its set of characters. A necessary

[45] See [H 2002] and the commentary by Ulrich Felgner, pp. 598–601.

condition for W is relatively easy to find, and Hausdorff succeeded in showing that this condition is also sufficient, that is, that for any W that satisfies this condition there will be an ordered set having W as its set of characters. For this purpose one requires a large reservoir of ordered sets, and this Hausdorff was able to create using his theory of general ordered products and powers.[46] In this reservoir one finds such interesting structures as Hausdorff's η_α normal types. Cantor had already regarded the type $\eta = \eta_0$ of the rational numbers in their natural ordering. He discovered that this type is universal with respect to the type class $T(\aleph_0)$ of all countable order types, that is, for every countable order type μ there exists a subset in η that has the type μ. Hausdorff's η_α type accomplishes the same thing for the type class $T(\aleph_\alpha)$. The question whether there exist $\eta_{\alpha+1}$ sets with the least possible cardinality $\aleph_{\alpha+1}$ then leads to the question whether $2^{\aleph_\alpha} = \aleph_{\alpha+1}$ holds. It was in this context that Hausdorff raised the generalized continuum hypothesis for the first time. His η_α sets were the point of departure for the notion of saturated structures, which has since played a major role in model theory.[47]

Hausdorff's general products and powers also led

[46] See [H 2002], pp. 604–605.

[47] On this see the essay by Ulrich Felgner: "Die Hausdorffsche Theorie der η_α-Mengen und ihre Wirkungsgeschichte". In: [H 2002], pp. 645–674.

him to the concept of partially ordered sets. Furthermore it turned out that the final gradations of sequences and functions he had been intensively studying were partial orderings. His proof of the existence of (ω_1, ω_1^*) gaps in the maximal ordered subsets of these semi-ordered sets is among Hausdorff's deepest results in set theory. Hausdorff was able to show that every ordered subset of a partially ordered set is contained in a maximal ordered subset by using the well-ordering theorem. This theorem is known today as Hausdorff's maximal chain theorem („Maximalkettensatz"). Not only does it follow from the well-ordering theorem (resp. the axiom of choice), it was later shown to be equivalent to both of them.[48]

Already in 1908 Arthur Schoenflies pointed out in the second part of his report on set theory that the more recent theory of ordered sets (that is the extensions of this theory undertaken after Cantor) were almost exclusively due to Hausdorff.[49] Schoenflies's conclusion suggests a more general comment bearing on the historiography of set theory, which until now has concentrated almost entirely on foundations issues, in particular discussions involving the axiom of choice, as well as the attempts in various ma-

[48] Regarding this theorem and similar results of Casimir Kuratowski and Max Zorn, see the commentary by U. Felgner in [H 2002], pp. 602–604.

[49] [S 1908], p. 40.

thematical and philosophical directions to overcome the antinomies. The extensions of set theory itself immediately after Cantor have, on the other hand, received comparatively little attention in the historical literature with the exception of the work of Zermelo; this applies in particular to the contributions of Hausdorff and Hessenberg.

In the summer semester of 1910 Hausdorff was appointed to a position as official associate professor (planmäßigen Extraordinarius) at Bonn University. As mentioned above, he found the academic atmosphere in Bonn far more to his liking than that in Leipzig. There he had not taught any courses in set theory since 1901, even though this was his primary field of research. After his arrival in Bonn, however, he immediately gave a course on set theory, which he repeated in the summer semester of 1912, though in a revised and expanded form. It was during that summer that he began work on his magnum opus, "Grundzüge der Mengenlehre". He completed it in Greifswald, where Hausdorff began teaching as a full professor (Ordinarius) in the summer semester of 1913; his book appeared in print in April 1914.

Set theory, as this area of mathematics was understood at the time, included not just the general theory of sets but also point sets as well as the theories of content and measure. Hausdorff's work was the first textbook that dealt systematically with all aspects of set theory in this comprehensive sense and which provided complete proofs in a masterful form. Moreover, it went well beyond the presentation of known results: it contained a number of significant original contributions by its author, which can only briefly be described here.

The first six chapters of the “Grundzüge” deal with general set theory. Hausdorff begins by setting out an algebra for sets that includes some new concepts that would prove influential (Differenzenketten, rings and fields of sets, δ- and σ-systems). These introductory paragraphs on sets and their operations also contain the modern set-theoretic concept of a function; here we encounter, so to speak, many of the ingredients that form the modern language of mathematics. There follows in chapters 3 to 5 the classical theory of cardinal numbers, order types, and ordinal numbers. In the sixth chapter on „Relations between ordered and well-ordered sets (Beziehungen zwischen geordneten und wohlgeordneten Mengen)“ Hausdorff presents, among other things, the most important results from his own researches on ordered sets.

The chapters on „point sets“ – one might prefer to say on topology – exude the spirit of a new era. Here Hausdorff presents for the time, beginning with his axioms for neighborhoods, a systematic theory of topological spaces, to which he added the separation axiom known today by his name. This theory arose through a comprehensive synthesis involving the work of other mathematicians as well as Hausdorff's own reflections on the space problem. The concepts and theorems from classical point set theory in $\mathbb{R}^n$ are now extended – so far as this is possible – to the general case, where they are subsumed into the newly created general or set-theoretic topology. Yet in the course of carrying out this „translation work," Hausdorff created a number of fundamentally new constructions for topology like the interior and closure operations, while developing the fundamental concepts of open set (which he called a „Gebiet") and compactness, a concept he took from Fréchet. He also established and developed the theory of connectedness, introducing in particular the notions of „components" and „quasi-components". He further specialized general topological spaces by means of the first and second Hausdorff countability axioms. The metric spaces comprise a large class of spaces that satisy the first countability axiom. These were introduced in 1906 by Fréchet, who called them „classes (E)"; the terminology „metrischer Raum" is due to Hausdorff. In his "Grundzüge" he gave a systematic presentation of the theory of metric spaces, to which he added several new concepts (Hausdorff metric, completion, total boundedness, ρ-connectedness, reducible sets). Fréchet's work[50] had received little attention; it was through Haus-

[50] [Fr 1906].

dorff's "Grundzüge" that metric spaces became widely familiar to mathematicians.[51]

Both the chapter on mappings as well as the final chapter of the "Grundzüge" on measure theory and integration are impressive for the generality of their approach and the originality of the presentation. Hausdorff's laconic remarks pointing to the significance of measure theory for probability would prove to be highly insightful. The final chapter also contains the first correct proof of the strong law of large numbers of Borel.[52] Finally, the appendix contains the single most spectacular result in the whole book, namely, Hausdorff's theorem that one cannot define a finitely additive measure (invariant under congruences) on all bounded subsets in $\mathbb{R}^n$ for $n \geq 3$. Hausdorff's proof is by means of a famous paradoxical decomposition of the sphere, for which it is necessary to invoke the axiom of choice.[53]

In the course of the twentieth century it became standard practice to place mathematical theories on a set-theoretic and axiomatic basis. The creation of

[51] Detailed commentaries on Hausdorff's contributions to general topology and the theory of metric spaces can be found in [H 2002], pp. 675–787.

[52] Shristi D. Chatterji gives commentary on measure theory and integration in the "Grundzüge" in [H 2002], pp. 788–800; see also [Ch 2002].

[53] On the historical impact of Hausdorff's sphere paradox, see [H 2001], pp. 11–18; see als the article by Peter Schreiber in [Br 1996], pp. 135–148, and the monograph [W 1993].

axiomatically grounded general theories, as for example in general topology, served among things to expose the structural elements common to various concrete situations or special areas and to place these in an abstract theory that subsumed all these as special cases. By so doing, there is a considerable gain in simplicity, unity, and ultimately in the economy of thought. Hausdorff gave special emphasis to this viewpoint in his "Grundzügen".[54] In this respect the topological chapters in the "Grundzüge" represent a pioneering achievement that paved the way for the development of modern mathematics. This modern conception of the essence of mathematics, made manifest through this new methodological orientation, had been conceived by Hausdorff many years before he composed the "Grundzüge", indeed well before the appearance of the relevant works of Fréchet and Friedrich Riesz.[55] An important impulse in this direction surely came from the "Grundlagen der Geometrie", which David Hilbert published in 1899. In Hausdorff's lecture course on "Time and Space", held during the winter semester of 1903/04, he remarked about mathematics in general: *Mathematics stands completely apart not only from the actual meaning that one attributes to its concepts but also from the actual validity one ascribes to its propositions. Its undefinable concepts are arbitrarily chosen objects of*

[54] [H 1914], p. 211.
[55] [Fr 1906], [Ri 1907, 1908].

thought, its axioms are also arbitrary, though chosen so as to be free from contradiction. Mathematics is a science of pure thought, just as is formal logic.[56] Hausdorff had this to say about space in particular: *Thus: space is a logical construction, namely it includes all propositions that follow logically from the arbitrarily chosen axioms, whereas the concepts employed are arbitrarily chosen objects of thought.*[57] When considering these quotations, one might wonder why Hausdorff did not undertake to secure the ultimate foundations, „das Fundament des Fundamentes" ("Grundzüge", p. 1) by developing set theory on an axiomatic basis. He was, of course, familiar with Zermelo's axiomatization, but he regarded this theory as only provisional: *By stipulating suitable conditions E. Zermelo undertook the [...] necessary attempt to curtail the processes leading to a boundless construction of sets. However, these highly astute investigations cannot yet be regarded as complete, and since an introduction to set theory along this path would surely lead to great difficulties for beginners, we prefer here to admit the naive concept of set, taking due account of the restrictions necessary in order to cut off the path leading to the paradoxes.*[58] It surely did not escape Hausdorff that Zermelo's concept of „definite property" (definiten

[56]NL Hausdorff: Kapsel 24: Fasz. 71, Bl. 4.

[57]Ibid., Bl. 31.

[58][H 1914], p. 2.

Eigenschaft) lacked precision.[59] In the remainder of the "Grundzüge" he avoided entering into foundational questions.[60]

The "Grundzüge der Mengenlehre" appeared at the dawning of the First World War. When it broke out in August 1914 scientific life in Europe was affected in the most dramatic ways. Under these circumstances, Hausdorff's book exerted hardly any impact for the next five to six years. After the war ended, a new generation of researchers began to take up the many suggestive impulses it contained, especially for topology, now a central field of interest. The reception of Hausdorff's ideas was enhanced by the founding in 1920 of a new journal in Poland, "Fundamenta Mathematicae". This was the first mathematical journal specializing in the fields of set theory, topology, the theory of real functions, measure theory and integration, functional analysis, logic, and the foundations of mathematics. Within this spectrum of interests, general topology occupied a central place. Hausdorff's "Grundzüge" was cited with great frequency beginning with the very first issue of "Fundamenta Mathematicae". In the 558 papers (exclu-

[59] On this, see [Fe 1979], pp. 3–8 und pp. 49–91.

[60] On these matters, see Peter Koepke: "Metamathematische Aspekte der Hausdorffschen Mengenlehre". In: [Br 1996], pp. 71–106. There one finds an interesting parallel between set-theoretic relativism and epistemological relativism in "Chaos in kosmischer Auslese".

ding the three written by Hausdorff himself) that appeared in the first twenty volumes between 1920 and 1933, no fewer than 88 referred to the "Grundzüge". Here one must also take account that Hausdorff's concepts had become so commonplace that one finds these in several papers in which he was not explicitly cited.

Hausdorff's "Grundzüge" had a similar influence on the Russian topological school founded by Paul Alexandroff and Paul Urysohn. This is evident from what remains in Hausdorff's Nachlaß from his correspondence with Alexandroff and Urysohn (after Urysohn's early death with Alexandroff alone) as well as from Urysohn's "Mémoire sur les multiplicités Cantoriennes" ([U 1925/1926]), a work the size of a book in which Urysohn set forth his theory of dimension, citing the "Grundzüge" no less than sixty times. The demand for Hausdorff's book continued until well after the Second World War, as attested by the three Chelsea reprints that appeared in 1949, 1965, and 1978.

In 1916 Hausdorff and Alexandroff solved (independently of one another) the continuum problem for Borel sets [61]: Every Borel set in a complete separable metric space is either at most countable or has

[61] [H 1916], [A 1916]. The notion of a „Borel set" in the modern sense was introduced by Hausdorff in the "Grundzüge". Schoenflies had used the term Borel sets merely for the case of G_δ-sets.

the power of the continuum. This result generalizes the theorem of Cantor-Bendixson, which makes the same assertion for closed subsets in $\mathbb{R}^n$. Earlier in 1903 William Henry Young extended this theorem to linear G_δ-sets[62] and in 1914 Hausdorff proved it for $G_{\delta\sigma\delta}$-sets in the "Grundzügen". The theorem of Alexandroff and Hausdorff proved to be a powerful impulse for the further development of descriptive set theory.[63]

Among Hausdorff's publications from his tenure in Greifswald, one in particular occupies a special place: his paper on dimension and outer measure ("Dimension und äußeres Maß").[64] This publication has remained highly relevant up to the present time and has probably been cited more often in recent years than any other research paper from the decade 1910 to 1920. Here a few technicalities are required: Let $\mathcal{U}$ be a system of bounded sets in $\mathbb{R}^q$ such that each set $A \subset \mathbb{R}^q$ can be covered by the union of at most countably many sets $U \in \mathcal{U}$ with diameters $d(U) < \varepsilon$ ($\varepsilon > 0$ arbitrary). Let $\lambda(x)$ be a continuous, strictly monoton increasing nonnegative function on $[0, \infty)$,

[62] [Y 1903].

[63] [AH 1935], p. 20. For further information see the commentary of Vladimir Kanovei and Peter Koepke in [H 2002], pp. 779–782 und in [H 2008], pp. 439–442.

[64] [H 1919a].

then for $A \subset \mathbb{R}^q$ Hausdorff introduces

$$L_\varepsilon^\lambda(A) = \inf \left\{ \sum_{n \geq 1} \lambda(d(U_n)) : \ A \subset \bigcup_{n \geq 1} U_n, \ d(U_n) < \varepsilon \right\}$$

$$\text{and} \qquad L^\lambda(A) = \lim_{\varepsilon \downarrow 0} L_\varepsilon^\lambda(A).$$

This $L^\lambda(A)$ is today called the Hausdorff measure for the function $\lambda(x)$. Hausdorff assigned to a set A the dimension $[\lambda]$ if

$$0 < L^\lambda(A) < \infty.$$

The fundamental and difficult question that now arises is the following: for a given function λ do there always exist sets $A \subset \mathbb{R}^q$ having dimension $[\lambda]$? Hausdorff was able to show that this is indeed so for every strictly monoton increasing, everywhere concave continuous function $\lambda(x) : [0, \infty) \to [0, \infty)$ with $\lambda(0) = 0$ and $\lim_{x \to \infty} \lambda(x) = \infty$. In the case where $\lambda(x) = x^p$, p positive real, one obtains the usual concepts associated with Hausdorff measure and Hausdorff dimension. The Hausdorff dimension of a set A is then the number α, for which

$$\alpha = \sup\{p > 0 : \ L^{(p)}(A) = \infty\} =$$
$$\inf\{p > 0 : \ L^{(p)}(A) = 0\},$$

where $L^{(p)} = L^\lambda$ and with $\lambda(x) = x^p$.

Hausdorff's concept of dimension is a finely tuned instrument for characterizing and comparing sets that are „highly jagged." The concepts in "Dimension und äußeres Maß" have been applied and further developed in numerous areas, for example, in the theory of dynamical systems, geometric measure theory, the theory of self-similar sets and fractals, the theory of stochastic processes, harmonic analysis, potential theory, and number theory.[65] Unfortunately the boom of interest in „fractal theory" has often led to misunderstandings and misinterpretations about Hausdorff's conceptions.[66]

The University of Greifswald was a small Prussian provincial university of merely local importance. Its mathematics institute was small, and in the summer semester of 1916 and the following winter semester Hausdorff was the only mathematician teaching in Greifswald! Due to this circumstance, his teaching activities were almost completely dominated by elementary courses. His situation improved markedly from a scientific standpoint when he went to Bonn in 1921. Here he had the opportunity to expand his

[65] On the historical impact of "Dimension und äußeres Maß" see the articles by Bandt/Haase and Bothe/Schmeling in [Br 1996], pp. 149–183 and pp. 229–252 as well as the commentary by Shristi D. Chatterji in [H 2001], pp. 44–54, and the literature cited therein.

[66] About this see Klaus Steffen: "Hausdorff-Dimension, reguläre Mengen und total irreguläre Mengen." In: [Br 1996], pp. 185–227.

teaching to a wide number of themes and to lecture over and again on his current research interests. Particularly noteworthy, for example, is the lecture course he offered in the summer semester of 1923 on probablity theory[67] in which he placed this theory on axiomatic and measure-theoretic foundations, already ten years before the publication of A. N. Kolmogoroff's "Grundbegriffe der Wahrscheinlichkeitsrechnung". In Bonn Hausdorff found in Eduard Study and later Otto Toeplitz colleagues who were not only outstanding mathematicians but who also became good friends.

During this second period in Bonn Hausdorff produced important work in analysis. In [H 1921] he developed an entire class of summation methods for divergent series which today are known as Hausdorff methods.[68] The classical methods of Hölder and Cesàro are special cases of these Hausdorff methods. Each such Hausdorff method is given by a sequence of moments; in this context Hausdorff gave an elegant solution of the problem of moments for a finite interval that bypasses the theory of continued fractions. In [H 1923b] he dealt with a special moment problem for a finite interval (subject to certain restrictions on the generating density $\varphi(x)$, for

[67] NL Hausdorff: Kapsel 21: Fasz. 64, reprinted in its entirety with detailed commentary in [H 2005], pp. 595–756.

[68] In Hardy's classical study [Har 1949] he devotes an entire chapter to Hausdorff methods.

example that $\varphi(x) \in L^p[0,1]$). Hausdorff spent many years working on criteria for the solvability and determination of moment problems, as evidenced by hundreds of pages left in his posthumous papers.[69] Hausdorff made a fundamental contribution to the emergence of functional analysis in the 1920s with his extension of the Fischer-Riesz theorem to L^p spaces in [H 1923 a]. There he also proved the inequalities named after him and W. H. Young:[70] If a_n are the Fourier coefficients of $f \in L^q(0,2\pi)$, $q \leq 2$, $\frac{1}{p}+\frac{1}{q}=1$, then

$$\left(\sum_{-\infty}^{\infty} |a_n|^p\right)^{\frac{1}{p}} \leq \left(\frac{1}{2\pi}\int_0^{2\pi} |f|^q\,dx\right)^{\frac{1}{q}}.$$

If $\sum_{-\infty}^{\infty} |a_n|^q$ converges, then there exists an $f \in L^p(0,2\pi)$ having these a_n as its Fourier coefficients, and furthermore

$$\left(\frac{1}{2\pi}\int_0^{2\pi} |f|^p\,dx\right)^{\frac{1}{p}} \leq \left(\sum_{-\infty}^{\infty} |a_n|^q\right)^{\frac{1}{q}}.$$

The Hausdorff-Young inequalities served as the point

[69] On the entire complex of these published and unpublished works, see [H 2001], pp. 105–171, 191–235, 255–267 and 339–373.

[70] Young had proved these for the special case $p = 2n$, $n = 2, 3, \ldots$.

of departure for wide ranging new developments.[71] In 1927 Hausdorff published his book "Mengenlehre", which has been declared as the second edition of the "Grundzüge". In reality this was a totally new book. In order to appear in the Göschen series, it was necessary to give a far more restricted presentation than in the "Grundzüge". Thus large parts of the theory of ordered sets and the sections on measure theory and integration had to be dropped. „Even more regrettable than these omissions" – according to Hausdorff in his preface – *was the need to save further room in point set theory by sacrificing the topological standpoint, despite its attractions for many readers of the first edition, and instead confining the discussion to the simpler theory of metric spaces, [...]*[72] In fact, some reviewers of the work expressly regretted this circumstance. As a form of compensation, however, Hausdorff offered an up-to-date presentation of the state of research in descriptive set theory. This insured that his new book received almost as strong a reception as had the "Grundzüge", especially in "Fundamenta Mathematicae". It became a highly popular textbook and appeared again in 1935 in an expanded second edition, which was reproduced by Dover in 1944. An English translation was published in 1957 with new printings in 1962,

[71] See the commentary by Shristi D. Chatterji in [H 2001], pp. 182–190.
[72] [H 1927], pp. 5–6.

1978 and 1991. A Russian edition came out in 1937, though this is not really a true translation; parts of the book were reworked by Alexandroff and Kolmogoroff in order to put the topological standpoint back in the foreground.[73]

In 1928 Hans Hahn wrote a review of the "Mengenlehre".[74] Possibly Hahn already sensed the dangers of German anti-Semitism when he ended his review with these words: *This in every respect masterful presentation of a difficult and hazardous subject is a work of the type written by those who have carried the fame of German science around the world, a work of which the author as well as all German mathematicians may be proud.*[75]

With the assumption of power by the National Socialists anti-Semitism became an official state doctrine. Hausdorff was not directly affected in 1933 by the notorious „law to restore the civil service" because he had already been a German civil servant since before 1914. His teaching activity was, however, apparently affected by activities undertaken by Nazi student functionaries. In his manuscript for his

[73] The complete text of the "Mengenlehre" is reprinted in [H 2008] (pp. 41–351). Background and reception to the work appear in an historical introduction (pp. 1–40), and the text itself receives detailed commentary (pp. 352–398) regarding mathematical as well as historical matters.

[74] Reprinted in [H 2008], pp. 416–417.

[75] [Ha 1928], p. 58.

lecture course “Infinitesimalrechnung III” held during the winter semester of 1934/35 he noted on page 16: „Interrupted 20 November“[76] Two days later, on 22 November 1934, the „Westdeutsche Beobachter“ reported in an article entitled „Party educates the Political Students“ that „during these days“ a working conference of the Nazi Student Union was taking place at Bonn University. The focus of their work during this semester was the theme of „race and folklore“. These circumstances make it likely that Hausdorff's decision to break off his lectures was connected with this political activity. At no other time in his long career, except for the brief period of the Kapp Putsch, did he ever cancel a lecture course.

On 31 March 1935, after some back and forth, Hausdorff retired as an emeritus professor in Bonn. For his forty years of successful labor in German higher education he received not a word of thanks from the then responsible authorities. He continued to work on indefatigably, publishing not only the newly revised version of his book “Mengenlehre” but also seven papers on topology and descriptive set theory, all of which appeared in two Polish journals: one paper in “Studia Mathematica”, the others in “Fundamenta Mathematicae”. Here we can only make a few brief remarks about this work; all of which is reprinted in volume III of the Gesammelten Werke ([H 2008])

[76]NL Hausdorff: Kapsel 19: Fasz. 59.

with detailed commentary.

In his final publication [H 1938], Hausdorff showed that a continuous mapping from a closed subset F of a metric space E can be extended to all of E (allowing for the possibility that the image space can also be extended). In particular, a homeomorphism defined on F can be extended to a homeomorphism on all of E. This work was a continuation of earlier investigations published in ([H 1919b] and [H 1930]). In [H 1919b] Hausdorff gave a new proof of the Tietze extension theorem, and in [H 1930] he showed the following: If E is a metric space and $F \subset E$ closed, and if on F is given a new metric that leaves the original topology invariant, then this new metric can be extended to the entire space without altering its topology. In [H 1935b] Hausdorff studied spaces that fulfill the Kuratowski closure axioms, except for the axiom demanding that the closure operation be idempotent. He called these „gestufte Räume“ (today they are usually known as closure spaces) and he used them to study relations between Fréchet's limit spaces and topological spaces.

The unpublished papers in Hausdorff's Nachlaß also show how he continued not only to work on but to follow the most recent developments in areas that interested him during these ever more difficult times. A major source of support for him came from Erich Bessel-Hagen, who remained a faithful friend of the Hausdorff family throughout their ordeal. Bessel-

Hagen brought books and journals from the mathematics library, which Hausdorff, as a Jew, was no longer allowed to enter.

Several articles would not suffice to name all the perfidious laws, decrees, ordinances, and other legalistic machinations designed to discriminate and isolate the Jews and deprive them of their property and rights. Historians have counted them though: up to the November 1938 pogrom there were more than 500 such proclamations. One wonders, why Hausdorff, an internationally recognized scholar living under such conditions, did not attempt to emigrate during the mid 1930s. The answer can only remain conjectural: in Bonn he had his home, his library and the possibility to work, some true friends, and although he was always a skeptic, even he would not have considered it possible that the Nazi regime would destroy the economic foundations established by elderly people in the course of their long lives and that ultimately they would pay with their lives. The November pogrom, which came to be known as the Night of the Broken Glass (Reichskristallnacht), with its open brutality made all this quite evident and clear. Hausdorff, now over 70, at last made an attempt to emigrate. Here is a passage from a letter written by Richard Courant on 10 February, 1939 to Hermann Weyl: *Dear Weyl, I just received the enclosed short and very touching letter from Professor Felix Hausdorff (which please return), who is seven-*

ty years old and whose wife is sixty-five years old. He certainly is a mathematician of very great merit and still quite active. He asks me whether it would be possible to find a research fellowship for him.[77] Weyl and John von Neumann provided letters of recommendation that were presumably sent to American institutions and colleagues. In Weyl's letter, he emphasized Hausdorff's many accomplishments and contributions to mathematics, calling him: „A man with a universal intellectual outlook, and a person of great culture and charm." These efforts of Weyl and von Neumann were, however, evidently unsuccessful.

From several sources, in particular the letters of Bessel-Hagen, we know that Hausdorff and his family were forced to undergo a number of humiliations, especially after November 1938.[78] In mid 1941 the Nazi government began to deport the Jews in Bonn to the monastery „Zur ewigen Anbetung" in Bonn-Endenich, from which the nuns had been expelled. From there they were then transported to the extermination camps in the east. In January 1942, Felix Hausdorff, his wife, and her sister Edith Pappen-

[77] Veblen Papers, Library of Congress, Container 31, folder Hausdorff. We thank Reinhard Siegmund-Schultze, Kristiansand, for making a copy of this letter available. He was unable to find Hausdorff's original letter.

[78] Neuenschwander, E.: "Felix Hausdorffs letzte Lebensjahre nach Dokumenten aus dem Bessel-Hagen-Nachlaß". In: [Br 1996], pp. 253–270.

heim, who lived with them, were ordered to resettle in the internment camp in Bonn-Endenich. On 26 January all three took their own lives with an overdosage of Veronal. Their last resting place is located in the cemetary in Bonn-Poppelsdorf.

Some of Bonn's Jewish citizens probably still had illusions about the camp in Endenich; Hausdorff had none. Erwin Neuenschwander found Hausdorff's farewell letter to the Jewish lawyer Hans Wollstein in the papers of Bessel-Hagen.[79], from which we cite the beginning and end:

Dear Friend Wollstein!

By the time you receive this letter, we three will have solved this problem in another way – the way you always tried to dissuade us from. The feeling of safety that you predicted would be ours once the difficulties of moving had been overcome has not come about at all. On the contrary:

Even Endenich

Is perhaps not yet the end (das Ende nich)!

What has happened to the Jews in the last months awakes justified anxiety in us that we will no longer be allowed to experience bearable conditions. After expressing his gratitude to friends, and with great composure formulating his last wishes regarding his funeral and last will, Hausdorff wrote further: *Excu-*

[79] NL Bessel-Hagen, Universitätsarchiv Bonn. For the first time printed in [Br 1992], p. 94; as facsimile in [Br 1996], pp. 265–267.

se us for causing you troubles even after death; I am convinced that you will do what you can (and that is perhaps not very much). Excuse us also for our desertion! We hope that you and all our friends will experience better times.

Your truly devoted,

Felix Hausdorff

It remains to add that this last wish of Hausdorff's was not fulfilled: the lawyer Wollstein was murdered in Auschwitz.

Hausdorff's library was sold by his son-in-law and sole heir Arthur König. His posthumous papers were preserved by a friend of the family, the Bonn Egyptologist Hans Bonnet, who later wrote about their further fate in [Bo 1967]. Hausdorff's papers *[...] were not yet saved, for in December 1944 a bomb explosion destroyed my house and the manuscripts were mired in rubble from a collapsed wall. I dug them out without being able to pay attention to their order and certainly without saving them all. Then in January 1945 I had to leave Bonn [...]. When I returned in the summer of 1946 almost all the furniture had disappeared, but the papers of Hausdorff were essentially intact. They were worthless for treasure hunters. Nevertheless, they suffered losses and the remaining scattered pages were mixed together more than ever. The once well-ordered cosmos had*

become a chaos.[80] All of this led to evident losses among the documents (for example, only very few letters have survived). The late Professor Günter Bergmann from Münster performed a great service by carefully ordering the surviving 25,978 pages of the Hausdorff Nachlaß, an enormous effort that took several years of meticulous work. In 1980 he transfered the now secure results over to the Bonn University library. Bergmann also published a some of the preserved papers in two facsimile volumes.[81]
The 26th of January, 1992 marked the fiftieth anniversary of Hausdorff's death. On this occasion, Professor Egbert Brieskorn took the initiative in preparing a special exhibit that awoke considerable interest in Hausdorff's career, and not only in mathematical circles. At the same time a memorial colloquium was held from which emerged the volume [Br 1996] cited several times above. Parallel with these activities, efforts began to launch an editorial project to publish Hausdorff's work. Professor Friedrich Hirzebruch took the initiative in creating a Hausdorff Commission with the Northern Rhine-Westphalian Academy of Sciences; this Hausdorff Commission was placed under the direction of Professor Reinhold Remmert and it began to undertake the necessary organizational steps to bring the project in motion. From the beginning of November

[80] [Bo 1967], p. 76 (152).
[81] [H 1969].

1993 to the end of 1995 the author catalogued the holdings in the Hausdorff Nachlaß. As a prerequisite for the editorial work that would follow it was necessary to produce a finding aid book describing the contents of all the documents.[82]

In November 1996, with the support of the Deutsche Forschungsgemeinschaft, work was ready to begin on preparing the Hausdorff-Edition for publication. It will ultimately contain reprints of all his published astronomical and mathematical works along with detailed commentary. Only selected portions of his unpublished works can appear in the edition; the manuscripts in his Nachlaß play an important part, however, in the commentaries. There was a consensus from the start that Hausdorff's literary and philsophical works, written under his pseudonym, should also be taken up in the edition along with commentaries. For this purpose it was necessary to bring together scholars representing a broad spectrum of expertise. This group of contributing editors, some of whom are still associated with the project, consists of 16 mathematicians, four historians of mathematics, two literary scholars, one philosopher, and one astronomer. Their nationalities are al-

[82]This Findbuch is accessible on internet under:
`www.aic.uni-wuppertal.de/fb7/hausdorff/findbuch.asp`
This resource can be used for searches also, for example to determine whether, and if so in which documents, a person or concept of interest happens to appear.

so diverse, representing Germany, Switzerland, Russia, the Czech Republic, and Austria. Beginning in January 2002 the Hausdorff-Edition has been taken over as an official project of the Nordrhein-Westfälischen Akademie; the editorial responsibility for the edition as a whole lies with Egbert Brieskorn, Friedrich Hirzebruch, Reinhold Remmert, Walter Purkert and Erhard Scholz. A particular difficulty – though also a special appeal – is the interdisciplinary character of the project. On the one hand, it is important to trace the influences Hausdorff's philosophical work had on his mathematics, especially those conceptions which bear on fundamental changes connected with the passage to mathematical modernity. On the other hand, in his philosophical and occasionally in the literary works one hears mathematical overtones, especially in his lyric poetry, though often these are sporadic and difficult to grasp.
The entire edition will appear in nine volumes structured as follows:

Band I : Biographie. Hausdorff als akademischer Lehrer. Arbeiten über geordnete Mengen

Band II : "Grundzüge der Mengenlehre" (1914)

Band III : "Mengenlehre" (1927, 1935). Arbeiten zur deskriptiven Mengenlehre und Topologie

Band IV : Analysis, Algebra und Zahlentheorie

Band V : Astronomie, Optik und Wahrscheinlichkeitstheorie

Band VI : Geometrie, Raum und Zeit

Band VII : Philosophisches Werk ("Sant' Ilario". "Das Chaos in kosmischer Auslese". Essays zu Nietzsche)

Band VIII : Literarisches Werk ("Ekstasen", "Der Arzt seiner Ehre", Essays)

Band IX : Korrespondenz

Springer-Verlag has taken on the responsibility of publishing the Hausdorff-Edition, and five of the volumes have already appeared, namely volumes IV (2001), II (2002), VII (2004), V (2005), III (2008). The publisher has provided the books with an appealing design, and each individual volume contains a complete list of Hausdorff's writings, including those he published under his pseudonym. A glance at this list of works shows that there are several not even mentioned in this essay, for example those on algebra (including the Baker- Campbell- Hausdorff formula) and other important contributions to analysis and topology. For these, we can only refer the interested reader to the five respective volumes mentioned above.

References

[A 1916] Alexandroff, P: *Sur la puissance des ensembles mesurables B.* Comptes rendus Acad. Sci. Paris **162** (1916), 323–325.

[AH 1935] Alexandroff, P.; Hopf, H.: *Topologie.* Springer-Verlag, Berlin 1935.

[BP 1987] Beckert, H.; Purkert, W. (Hrsg.): *Leipziger mathematische Antrittsvorlesungen.* Teubner-Archiv zur Mathematik, Band 8, Leipzig 1987.

[Bl 1921] Blumberg, H.: *Hausdorff's Grundzüge der Mengenlehre.* Bull. of the AMS **27** (1921), 116–129. WA: [H 2002], 844–853.

[Bo 1967] Bonnet, H.: *Geleitwort.* Jahresbericht der DMV **69** (1967), 75 (151)–76(152).

[Br 1992] Brieskorn, E.: *Felix Hausdorff – Elemente einer Biographie.* In: *Katalog der Ausstellung „Felix Hausdorff – Paul Mongré (1868–1942)"*, Mathematisches Institut der Universität Bonn, 24. Januar bis 28. Februar 1992.

[Br 1996] Brieskorn, E. (Hrsg.): *Felix Hausdorff zum Gedächtnis. Aspekte seines Werkes.* Vieweg, Braunschweig/ Wiesbaden 1996.

[Br 1997] Brieskorn, E.: *Gustav Landauer und der Mathematiker Felix Hausdorff.* In: H. Delf; G. Mattenklott (Ed.): *Gustav Landauer im Gespräch*

– *Symposium zum 125. Geburtstag.* Tübingen 1997, S. 105–128.

[Ch 2002] Chatterji, S. D.: *Hausdorff als Maßtheoretiker.* Math. Semesterberichte **49** (2002), 129–143.

[Ch 2007] Chatterji, S. D.: *The Central Limit Theorem à la Hausdorff.* Expositiones Math. **25** (2007), 215–234.

[D 1967] Dierkesmann, M.: *Felix Hausdorff. Ein Lebensbild.* Jahresbericht der DMV **69** (1967), 51(127)–54(130).

[Eb 2007] Ebbinghaus, H.-D.: *Ernst Zermelo. An approach to his life and work.* Springer, Berlin etc. 2007.

[ET 1994] Eichhorn, E.; Thiele, E.-J.: *Vorlesungen zum Gedenken an Felix Hausdorff.* Heldermann Verlag, Berlin 1994.

[F 1948] Fechter, P.: *Menschen und Zeiten.* Bertelsmann, Gütersloh 1948.

[Fe 1979] Felgner, U. (Hrsg.): *Mengenlehre.* Wiss. Buchges., Darmstadt 1979.

[Fr 1906] Fréchet, M.: *Sur queques points du calcul fonctionnel.* Rendiconti del Circolo Mat. di Palermo **22** (1906), 1–74.

[H 1891] Hausdorff, F.: *Zur Theorie der astronomischen Strahlenbrechung* (Dissertation). Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Classe **43** (1891), 481–566.

[H 1895] Hausdorff, F.: *Über die Absorption des Lichtes in der Atmosphäre* (Habilitationsschrift). Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.- phys. Classe **47** (1895), 401–482.

[H 1896] Hausdorff, F.: *Infinitesimale Abbildungen der Optik*. Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Classe **48** (1896), 79–130.

[H 1897a] Hausdorff, F. (P. Mongré): *Sant' Ilario – Gedanken aus der Landschaft Zarathustras.* Verlag C. G. Naumann, Leipzig. VIII + 379 S.

[H 1897b] Hausdorff, F. (P. Mongré):*Sant' Ilario – Gedanken aus der Landschaft Zarathustras.* Selbstanzeige, Die Zukunft, 20.11.1897, 361.

[H 1897c] Hausdorff, F.: Das Risico bei Zufallsspielen. Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Classe **49** (1897), 497–548.

[H 1898] Hausdorff, F. (P. Mongré):*Das Chaos in kosmischer Auslese – Ein erkenntniskritischer*

Versuch. Verlag C. G. Naumann, Leipzig. VI und 213 S.

[H 1899] Hausdorff, F.: *Analytische Beiträge zur nichteuklidischen Geometrie.* Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Classe **51** (1899), 161–214.

[H 1900a] Hausdorff, F. (P. Mongré): *Ekstasen.* Gedichtband. Verlag H. Seemann Nachf., Leipzig. 216 S.

[H 1900b] Hausdorff, F.: *Zur Theorie der Systeme komplexer Zahlen.* Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Classe **52** (1900), 43–61.

[H 1901a] Hausdorff, F.: *Über eine gewisse Art geordneter Mengen.* Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Classe **53** (1901), 460–475.

[H 1901b] Hausdorff, F.: *Beiträge zur Wahrscheinlichkeitsrechnung.* Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Classe **53** (1901), 152–178.

[H 1902] Hausdorff, F. (P. Mongré):*Der Wille zur Macht.* Neue Deutsche Rundschau (Freie Bühne) **13** (12) (1902), 1334–1338.

[H 1903] Hausdorff, F.: *Das Raumproblem* (Antrittsvorlesung an der Universität Leipzig, gehalten am 4.7.1903). Ostwalds Annalen der Naturphilosophie **3** (1903), 1–23.

[H 1904a] Hausdorff, F. (P. Mongré):*Der Arzt seiner Ehre, Groteske.* Die neue Rundschau (Freie Bühne) **15** (8), (1904), 989–1013. Neuherausgabe als: *Der Arzt seiner Ehre. Komödie in einem Akt mit einem Epilog.* Mit 7 Bildnissen, Holzschnitte von Hans Alexander Müller nach Zeichnungen von Walter Tiemann, 10 Bl., 71 S. Fünfte ordentliche Veröffentlichung des Leipziger Bibliophilen-Abends, Leipzig 1910. Neudruck: S.Fischer, Berlin 1912, 88 S.

[H 1904b] Hausdorff, F.: *Der Potenzbegriff in der Mengenlehre.* Jahresbericht der DMV **13** (1904), 569–571.

[H 1906] Hausdorff, F.: *Untersuchungen über Ordnungstypen I, II, III.* Ber. Über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Klasse **58** (1906), 106–169.

[H 1907a] Hausdorff, F.: *Untersuchungen über Ordnungstypen IV, V.* Ber. über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Klasse **59** (1907), 84–159.

[H 1907b] Hausdorff, F.: *Über dichte Ordnungstypen.* Jahresbericht der DMV **16** (1907), 541–546.

[H 1908] Hausdorff, F.: *Grundzüge einer Theorie der geordneten Mengen.* Math. Annalen **65** (1908), 435–505.

[H 1909] Hausdorff, F.: *Die Graduierung nach dem Endverlauf.* Abhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig. Math.-phys. Klasse **31** (1909), 295–334.

[H 1914] Hausdorff, F.: *Grundzüge der Mengenlehre.* Verlag Veit & Co, Leipzig. 476 S. mit 53 Figuren. Nachdrucke: Chelsea Pub. Co. 1949, 1965, 1978.

[H 1916] Hausdorff, F.: *Die Mächtigkeit der Borelschen Mengen.* Math. Annalen **77** (1916), 430–437.

[H 1919a] Hausdorff, F.: *Dimension und äußeres Maß.* Math. Annalen **79** (1919), 157–179.

[H 1919b] Hausdorff, F.: *Über halbstetige Funktionen und deren Verallgemeinerung.* Math. Zeitschrift **5** (1919), 292–309.

[H 1921] Hausdorff, F.: *Summationsmethoden und Momentfolgen I, II.* Math. Zeitschrift **9** (1921), I: 74–109, II: 280–299.

[H 1923a] Hausdorff, F.: *Eine Ausdehnung des Parsevalschen Satzes über Fourierreihen.* Math. Zeitschrift **16** (1923), 163–169.

[H 1923b] Hausdorff, F.: *Momentprobleme für ein endliches Intervall.* Math. Zeitschrift **16** (1923), 220–248.

[H 1927] Hausdorff, F.: *Mengenlehre,* zweite, neubearbeitete Auflage. Verlag Walter de Gruyter & Co., Berlin. 285 S. mit 12 Figuren.

[H 1930] Hausdorff, F.: *Erweiterung einer Homöomorphie.* Fundamenta Mathematicae **16** (1930), 353–360.

[H 1935a] Hausdorff, F.: *Mengenlehre*, dritte Auflage. Mit einem zusätzlichen Kapitel und einigen Nachträgen. Verlag Walter de Gruyter & Co., Berlin. 307 S. mit 12 Figuren. Nachdruck: Dover Pub. New York, 1944. Englische Ausgabe: Set theory. Übersetzung aus dem Deutschen von J.R.Aumann et al. Chelsea Pub. Co., New York 1957, 1962, 1978, 1991.

[H 1935b] Hausdorff, F.: *Gestufte Räume.* Fundamenta Mathematicae **25** (1935), 486–502.

[H 1938] Hausdorff, F.: *Erweiterung einer stetigen Abbildung.* Fundamenta Mathematica **30** (1938), 40–47.

[H 1969] Hausdorff, F.: *Nachgelassene Schriften.* 2 Bd. Hrsg.: G. Bergmann, Teubner, Stuttgart 1969. Band I enthält aus dem Nachlaß die Faszikel 510–543, 545–559, 561–577, Band II die Faszikel 578–584, 598–658 (alle Faszikel sind im Faksimiledruck wiedergegeben).

[H 2001] Hausdorff, F.: *Gesammelte Werke. Band IV: Analysis, Algebra und Zahlentheorie.* Springer-Verlag, Berlin, Heidelberg etc., 2001.

[H 2002] Hausdorff, F.: *Gesammelte Werke. Band II: „Grundzüge der Mengenlehre“.* Springer-Verlag, Berlin, Heidelberg etc. 2002.

[H 2004] Hausdorff, F.: *Gesammelte Werke. Band VII: Philosophisches Werk.* Springer-Verlag, Berlin, Heidelberg etc. 2004.

[H 2005] Hausdorff, F.: *Gesammelte Werke. Band V: Astronomie, Optik und Wahrscheinlichkeitstheorie.* Springer-Verlag, Berlin, Heidelberg etc. 2005.

[H 2008] Hausdorff, F.: *Gesammelte Werke. Band III: Deskriptive Mengenlehre und Topologie.* Springer-Verlag, Berlin, Heidelberg etc. 2008.

[Ha 1928] Hahn, H.: *F. Hausdorff, Mengenlehre.* Monatshefte für Mathematik und Physik **35** (1928), 56–58.

[Har 1949] Hardy, G. H.: *Divergent Series.* Oxford Univ. Press, Oxford 1949.

[JN 1887] *Jahresbericht des Nicolai-Gymnasiums für das Jahr 1887.* Stadtarchiv Leipzig, Bestand Nicolai-Gymnasium.

[L 1967] Lorentz, G. G.: *Das mathematische Werk von Felix Hausdorff.* Jahresbericht der DMV **69** (1967), 54 (130)–62 (138).

[Pu 2004] Purkert, W.: *Kontinuumproblem und Wohlordnung – die spektakulären Ereignisse auf dem Internationalen Mathematikerkongreß 1904 in Heidelberg.* In: Seising, R.; Folkerts, M.; Hashagen, U. (Hrsg.): *Form, Zahl, Ordnung*, Boethius Bd. 48, Franz Steiner Verlag, Wiesbaden 2004, S. 223–241.

[Ri 1907] Riesz, F.: *Die Genesis des Raumbegriffs.* Math. und Naturwiss. Berichte aus Ungarn **24** (1907), 309–353.

[Ri 1908] Riesz, F.: *Stetigkeitsbegriff und abstrakte Mengenlehre.* Atti del Congr. Internaz. dei Mat., Roma **2** (1908), 18–24.

[S 1908] Schoenflies, A.: *Die Entwickelung der Lehre von den Punktmannigfaltigkeiten.* Teil II. Jahresbericht der DMV, 2. Ergänzungsband, Teubner, Leipzig 1908.

[St 2002] Stegmaier, W.: *Ein Mathematiker in der Landschaft Zarathustras. Felix Hausdorff als Philosoph.* Nietzsche-Studien **31** (2002), 195–240.

[U 1925/1926] Urysohn, P.: *Mémoire sur les multiplicités Cantoriennes.* Fundamenta Math. **7** (1925), 30–137; **8** (1926), 225–351.

[V 2000] Vollhardt, F.: *Von der Sozialgeschichte zur Kulturwissenschaft? Die literarisch-essayistischen Schriften des Mathematikers Felix Hausdorff (1868–1942): Vorläufige Bemerkungen in systematischer Absicht.* In: Huber, M.; Lauer, G. (Hrsg.): *Nach der Sozialgeschichte – Konzepte für eine Literaturwissenschaft zwischen Historischer Anthropologie, Kulturgeschichte und Medientheorie.* Max Niemeier Verlag, Tübingen 2000, S. 551–573.

[W 1993] Wagon, S.: *The Banach-Tarski Paradox.* Cambridge Univ. Press, Cambridge 1993.

[Y 1903] Young, W. H.: *Zur Lehre der nicht abgeschlossenen Punktmengen.* Berichte über die Verhandlungen der Königl. Sächs. Ges. der Wiss. zu Leipzig, Math.-Phys. Klasse **55** (1903), 287–293.